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# Geometric Adaptive Control of Attitude Dynamics on SO(3) with State Inequality Constraints

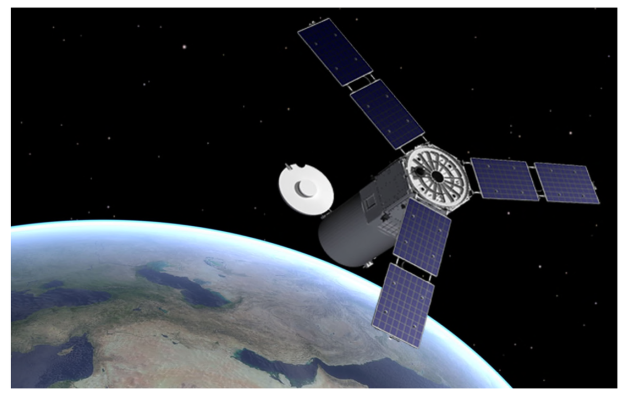

Shankar Kulumani and Christopher Poole

Flight Dynamics and Controls Laboratory (Dr. Taeyoung Lee)
Department of Mechanical and Aerospace Engineering, School of Engineering and Applied Science


## Background and Motivation

- Autonomous control of vehicles is critical for missions
  - Typical operations require extensive planning and human interaction
  - Vehicles must operate safely in hazardous environments
  - Applicable to under-water, aerial, and spacecraft scenarios
- Key technology for autonomy is large angle reorientations in the presence obstacles
  - Spacecraft have sensitive payloads e.g. optical sensors
  - Reorient while not pointing in dangerous directions e.g. Sun, Moon


Hubble


Agile S/C

- Problem: reorient a vehicle while avoiding certain directions
  - Sensor exclusion zone around the Sun
  - UAVs manuevering in restricted and congested locations
  - Laser emitters on industrial robots

## Spacecraft Orientation

- Rigid body attitude dynamics is a classic problem
- Configuration manifold is curved and nonlinear
  - Dynamics evolve on the Special Orthogonal Group: SO(3)
  - Unique properties: cannot be represented as a linear vector space
- Previous work is based on reduced attitude representations
  - Euler angles: 24 possible combinations which suffer singularities
  - Quaternions: no singularities but double cover SO(3)
- Geometric control: the development of control systems for systems evolving on nonlinear manifolds
  - Many systems cannot be defined correctly on Euclidean spaces
  - Innovative techniques avoid ambiguities and local coordinates and exactly describe the evolution of the system

## Attitude Dynamics

- Spacecraft is modeled as a rigid body rotating about its center of mass described by the Special Orthogonal Group

$$\mathrm{SO}(3) = \left\{ R \in \mathbb{R}^{3\times 3} \,|\, R^T R = I, \det R = 1 \right\}$$

- Euler's equations of motion govern the dynamics of a rigid body

$$J\dot{\Omega} + \Omega \times J\Omega = u + W(R,\Omega)\Delta,$$
$$\dot{R} = R\hat{\Omega},$$

- $R \in \mathrm{SO}(3)$ defines the orientation of the spacecraft with respect to an inertial reference frame
- $W(R,\Omega)\Delta$ models a wide range of external disturbances
  - Solar radiation pressure (SRP)
  - Gravity gradient moment
  - Air turbulence and gusts
  - Unknown mass distribution

## Configuration Error Function on SO(3)

- Constraint is defined in terms of unit-vectors on the two-sphere:

$$\mathrm{S}^2 = \left\{ q \in \mathbb{R}^3 \,|\, \|q\| = 1 \right\}$$

- We wish to avoid pointing spacecraft in a particular direction
  - Sensitive optical sensor - $r \in \mathrm{S}^2$ defines the sensor direction
  - Constraint direction - $v \in \mathrm{S}^2$ defines direction to distant object
- Hard cone constraint - strictly avoid pointing sensor towards the celestial object

$$r^T R^T v \leq \cos\theta$$

- Smooth, positive definite function which measures the error between current and desired configuration
- Error function is the product of two terms

$$\Psi(R) = A(R)B(R)$$

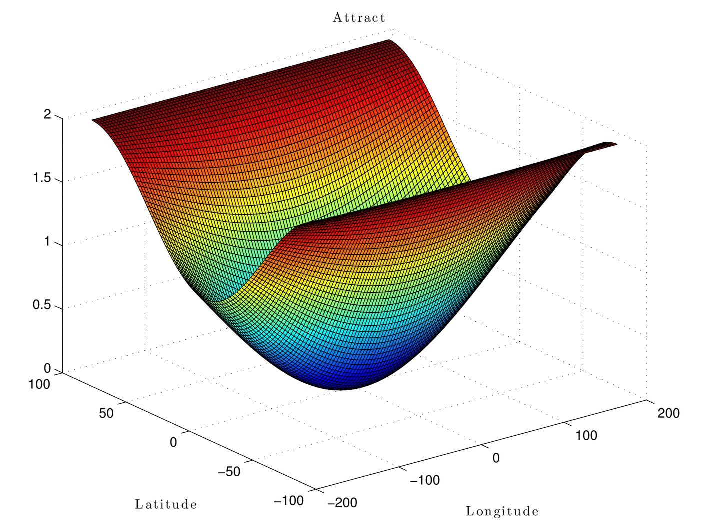
  - Attractive - drives system towards desired attitude

$$A(R) = \frac{1}{2}\mathrm{tr}\left[G\left(I - R_d^T R\right)\right]$$

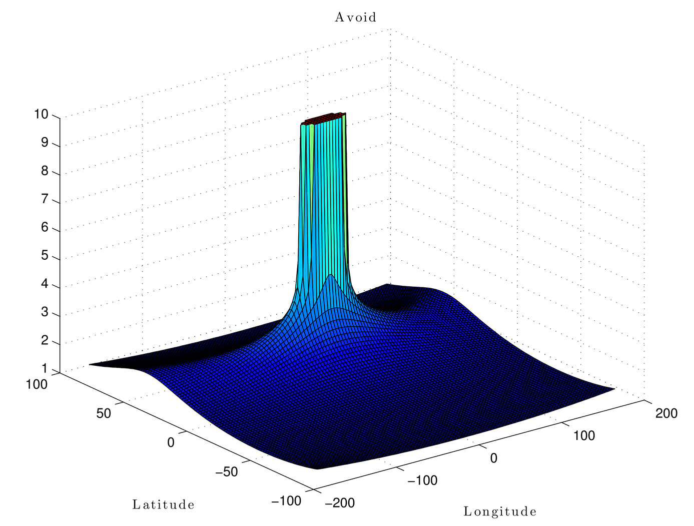
  - Repulsive - forces system away from constraint directions

$$B(R) = 1 - \frac{1}{\alpha}\ln\left(\frac{\cos\theta - r^T R^T v}{1 + \cos\theta}\right)$$

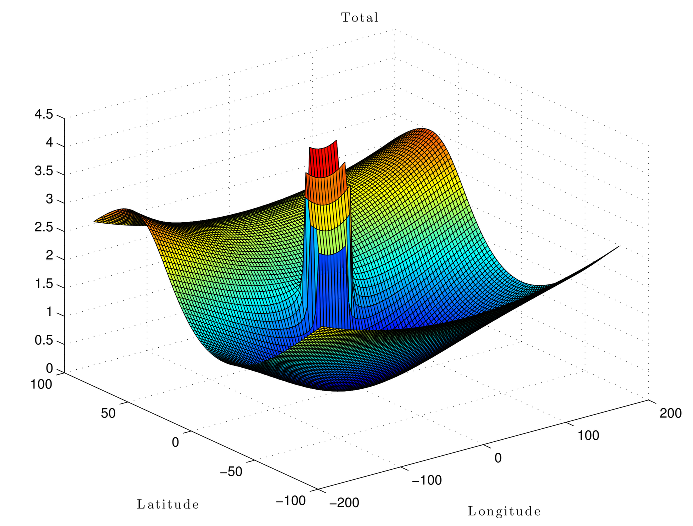
- Logarithmic barrier function causes the error to grow as $r^T R^T v \to \cos\theta$
  - $B(R) \to \infty$ as the constraint boundary is neared $r^T R^T v \to \cos\theta$
  - $B(R)$ has little impact on $\Psi$ when far from constraint as the logarithmic function quickly decays



Attractive $A(R)$



Repulsive $B(R)$



Configuration $\Psi$

- We can easily generalize this technique to an arbitrary number of constraints

$$\Psi = A\left[1 + \sum_i C_i\right] \quad \text{where } C_i = B - 1$$

- Lyapunov analysis is used to derive an adaptive control scheme which guarantees stability in the face of disturbances and obstacles

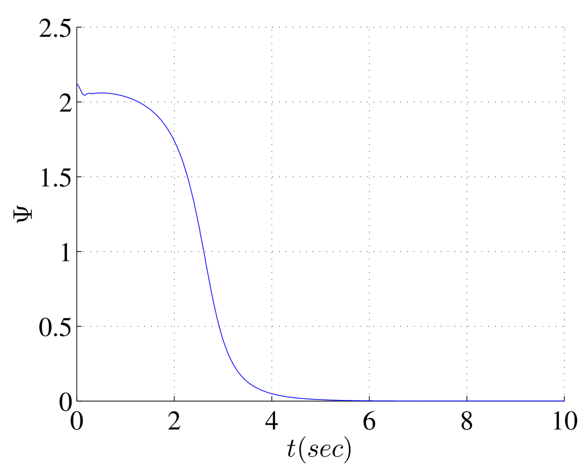
$$u = -k_R e_R - k_\Omega e_\Omega + \Omega \times J\Omega - W\bar{\Delta}$$
$$\dot{\bar{\Delta}} = k_\Delta W^T\left(e_\Omega + c e_R\right)$$

## Numerical Simulation

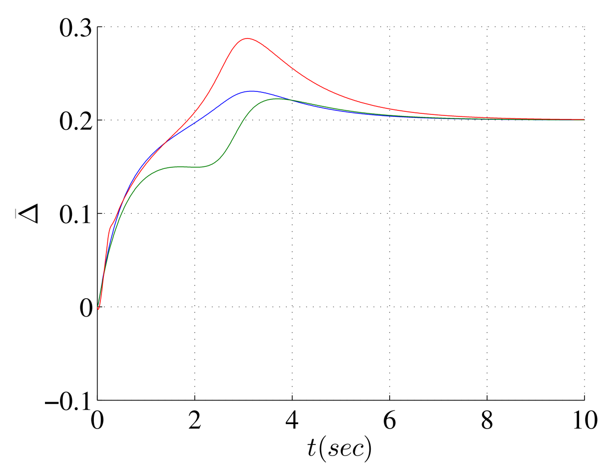
- Geometric Adaptive Controller is able to stabilize the rigid body while avoiding multiple constraints with a fixed but unknown external disturbance

$$\text{Initial: } R_0 = \exp(225^\circ \times \frac{\pi}{180}\hat{e}_3) \quad \text{Final: } R_d = I \quad \text{Disturbance: } \Delta = \left[0.2\ 0.2\ 0.2\right]^T$$

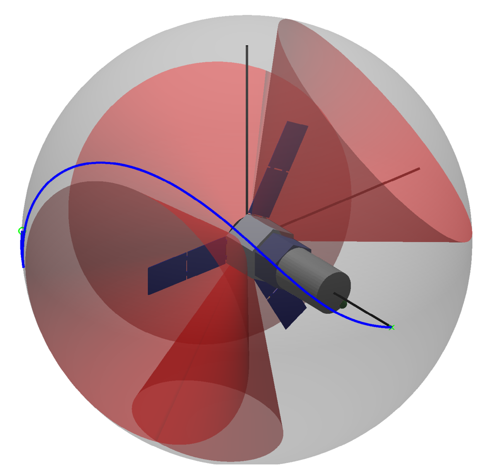
- The adaptive controller accurately accounts for the disturbance and ensures all constraints are satisfied



Configuration error $\Psi$

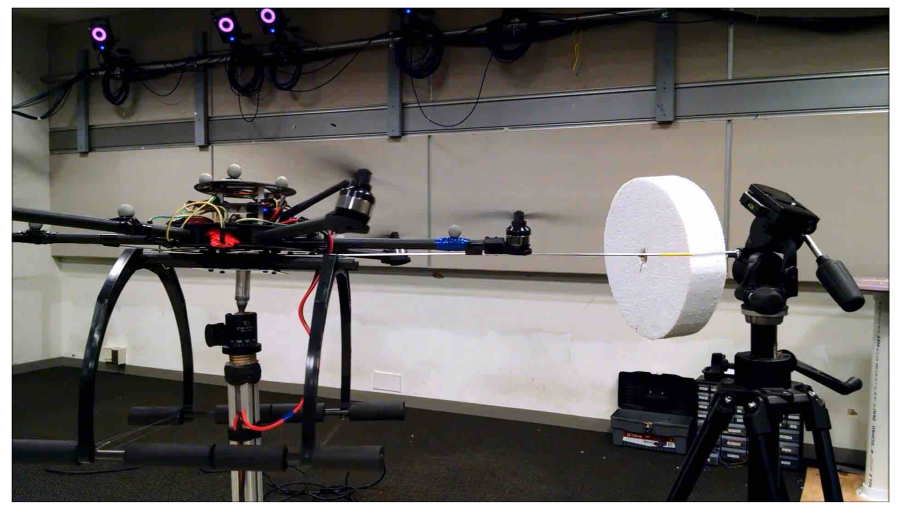


Disturbance estimate $\bar{\Delta}$


Attitude trajectory

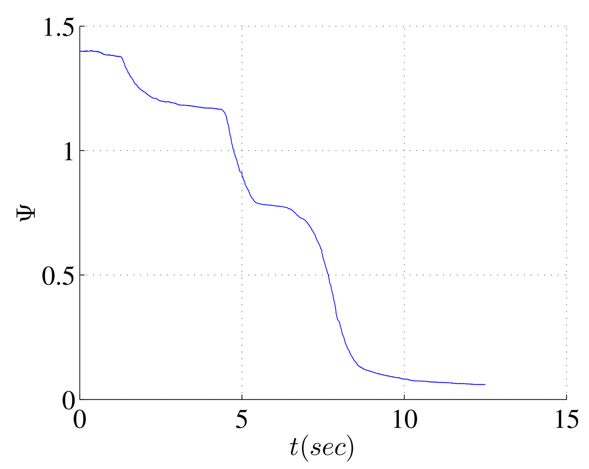
## UAV Validation

- Hexrotor UAV developed by the Flight Dynamics and Controls Laboratory
  - Three pairs of counter-rotating propellers
  - Attached to a spherical joint to emulate a fully actuated rigid body
  - Onboard computer module receives measurements from Vicon motion capture system and computes control input in real-time

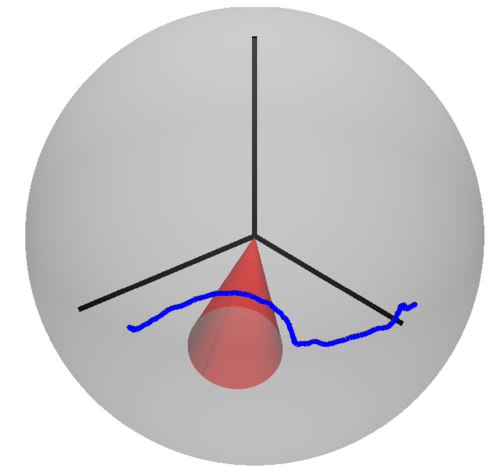

Attitude control testbed

- Hexrotor rotates about vertical axis while automatically avoiding the obstacle

$$\text{Initial: } R_0 = \exp(\frac{\pi}{2}\hat{e}_3) \quad \text{Final: } R_d = I$$



Configuration error $\Psi$


Attitude Trajectory

- Adaptive controller is robust to uncertainties and disturbances

## Conclusion

- Constrained geometric adaptive controller on SO(3)
  - Completely avoids singularities and ambiguities
  - Geometrically exact and conceptually simple attitude controller
  - Automatically satisfies multiple constraints without added complexity
- Obstacle avoidance computed in real-time with on-board software
  - Typical planning methods are only able to determine an obstacle-free path after multiple iterations and extensive computation
  - Large computation costs limit these methods to a priori calculation and make responsive control impossible
  - Randomized search algorithms can only offer a stochastic guarantee of convergence as the computation time increases
- Our control system is capable of handling any number of obstacles and offers a rigorous stability proof
  - Ideal for challenging scenarios with multiple obstacles or an environment which requires complex control
  - Computationally efficient and ideal for embedded systems with energy or computation limitations
  - Stability proof ensures manuvers always satisfy pointing constraints