Spacecraft Trajectory Design Near Asteroid 4769 Castalia

THE GEORGE WASHINGTON UNIVERSITY
WASHINGTON, DC

Shankar Kulumani

Flight Dynamics and Controls Laboratory (Dr. Taeyoung Lee)
Department of Mechanical and Aerospace Engineering, School of Engineering and Applied Science

Introduction

- Asteroids and comets are of significant interest
  - Science - Insight into early solar system formation
  - Mining - vast quantities of useful materials
  - Impact - high risk from hazardous Near-Earth asteroids
- Near-Earth asteroids (NEAs) are especially interesting
  - Orbit close to the Earth and are easily accessible
  - Many asteroids hold vast quantities of useful materials
  - Asteroid mining: Precious metals, propulsion fuels, semiconductors
  - Commercialization is feasible with huge amounts of possible profit
- High probability of future asteroid impacts

Asteroid Mining

Asteroid Impact

Technical Challenges

- Low-thrust propulsion systems offer innovative options
  - Electric propulsion offers much greater efficiency
  - Allows for greater velocity change with a reduced mass cost
  - Key component for long duration missions with frequent thrusting
  - Requires new methods of design
- Optimal trajectory design is complicated
  - Highly nonlinear and chaotic dynamics requires intuition by designer
  - Using low-thrust propulsion adds additional difficulties in accurately capturing the small perturbations
- Astrodynamic trajectory design typically uses direct optimal control
  - Large nonlinear programming problem inherently approximates the true optimal solution
  - High dimensionality of the solution makes it extremely computationally intensive

Gravitational Modeling

- Asteroids are extended bodies - not point masses
  - Gravity is the key force in orbital mechanics
  - An accurate representation of gravity is critical to accurate and realistic analysis
- Spherical Harmonic approach is popular but not ideal
  - Model is only valid outside of circumscribing sphere
  - Composed of an infinite series - always results in an approximation
  - Model will diverge when close to the surface and is not ideal for landing missions
- Polyhedron Gravitational model used to represent the asteroid
  - Gravity is a function of the shape model
  - Globally valid and closed-form analytical solution for gravity
  - Exact potential assumes a constant density assumption
  - Accuracy is only dependent on the shape

$$U(r)=\frac{1}{2}G\sigma\sum_{e\in\text{edges}} r_e\cdot E_e\cdot r_e\cdot L_e-\frac{1}{2}G\sigma\sum_{f\in\text{faces}} r_f\cdot F_f\cdot r_f\cdot\omega_f$$

Dynamics about the asteroid 4769 Castalia

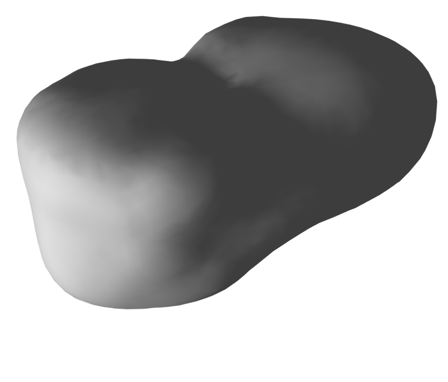
- Dynamics are very similar to the famous three-body problem

$$\begin{bmatrix}\dot{r}\\ \dot{v}\end{bmatrix}=\begin{bmatrix}v\\ g(r)+h(v)+u\end{bmatrix}$$

- Huge history of analytical tools allow for great insight into the dynamics
- Analytical insight is critical to understanding the free motion around an asteroid
  - We require an accurate understanding of the motion under the influence of gravity alone
  - Efficient use of the limited oboard fuel is dependent on exploiting the natural dynamics of the asteroid environment
- Jacobi Integral - single constant of motion which bounds the feasible regions in terms of "energy"

$$J(r,v)=\frac{1}{2}\omega^2\left(x^2+y^2\right)+U(r)-\frac{1}{2}\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right)$$

- Spacecraft is operating around 4769 Castalia
  - Discovered in 1989, Castalia is a potentially hazardous asteroid and passes close to the Earth
  - In 1989, Castalia passed close enough to allow for high resolution radar imagery
  - High resolution shape is used in polyhedral gravity model



Asteroid 4769 Castalia

Simulation Results

- Transfer between two periodic orbits of 4769 Castalia
  - Thruster represents a current electric propulsion $\approx 600\,\mathrm{mN}$
  - Combining multiple iterations of the rechability computation allows for general transfers
- Combining four iterations of the reachability set
  - Each iteration of the reachability set enlarges the achievable states
  - We choose a direction on the reachability set which lies closest to the target

$$d=\sqrt{k_x\left(x_f-x_t\right)^2+k_z\left(z_f-z_t\right)^2+k_{\dot{x}}\left(\dot{x}_f-\dot{x}_t\right)^2+k_{\dot{z}}\left(\dot{z}_f-\dot{z}_t\right)^2}$$

  - This iterative approach avoids the difficulty in choosing accurate initial guesses for optimization

- Optimal Control is used to calculate the reachability set

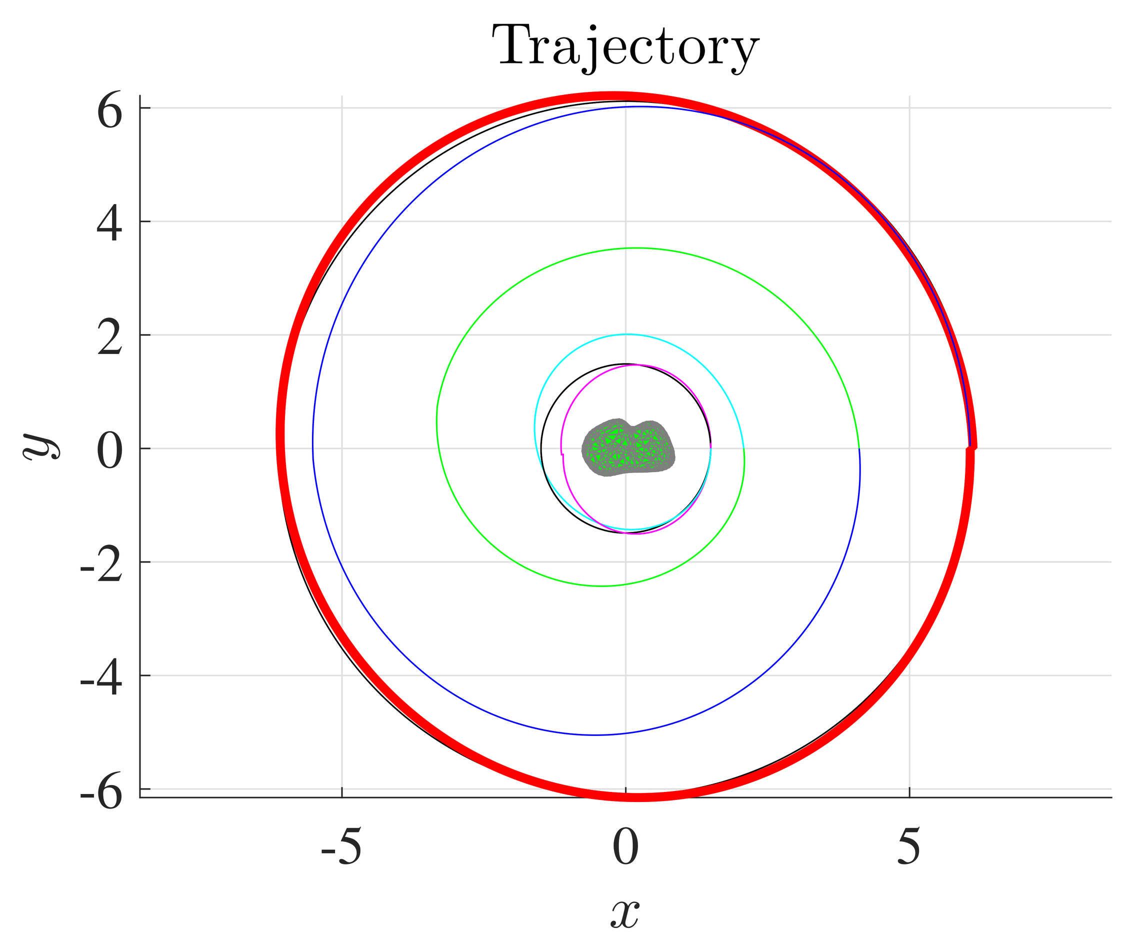
$$J=-\frac{1}{2}\left(x(t_f)-x_n(t_f)\right)^T Q\left(x(t_f)-x_n(t_f)\right)$$

- Maximize the distance on the section using the low thrust propulsion
- Thruster magnitude is limited by physical system

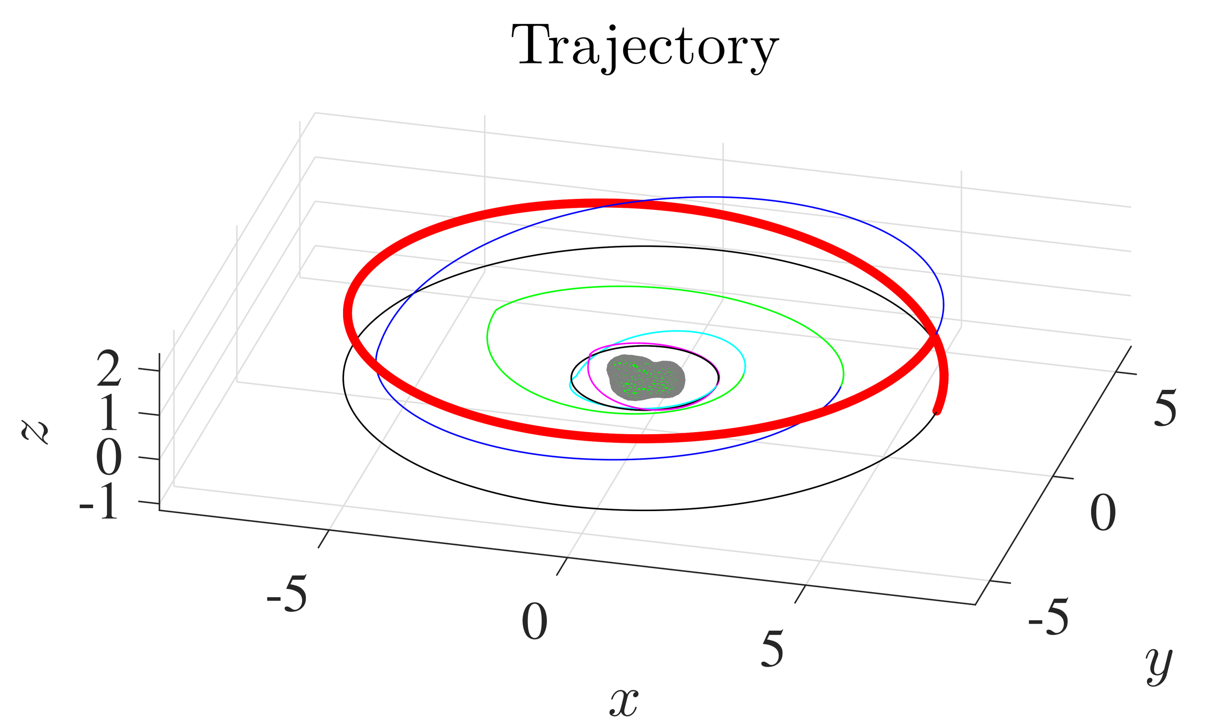
$$c(u)=u^Tu-u_m^2\leq 0$$

- Terminal constraints ensure intersection with the section

$$m_1=y=0$$
$$m_2=\left(\sin\phi_{1_d}\right)\left(x_1^2+x_2^2+x_3^2+x_4^2\right)-x_1^2=0$$
$$m_3=\left(\sin\phi_{2_d}\right)\left(x_2^2+x_3^2+x_4^2\right)-x_2^2=0$$
$$m_4=\left(\sin\phi_{3_d}\right)\left(2x_3^2+2x_3\sqrt{x_4^2+2x_4^2}\right)-x_3-\sqrt{x_4^2+x_3^2}=0$$

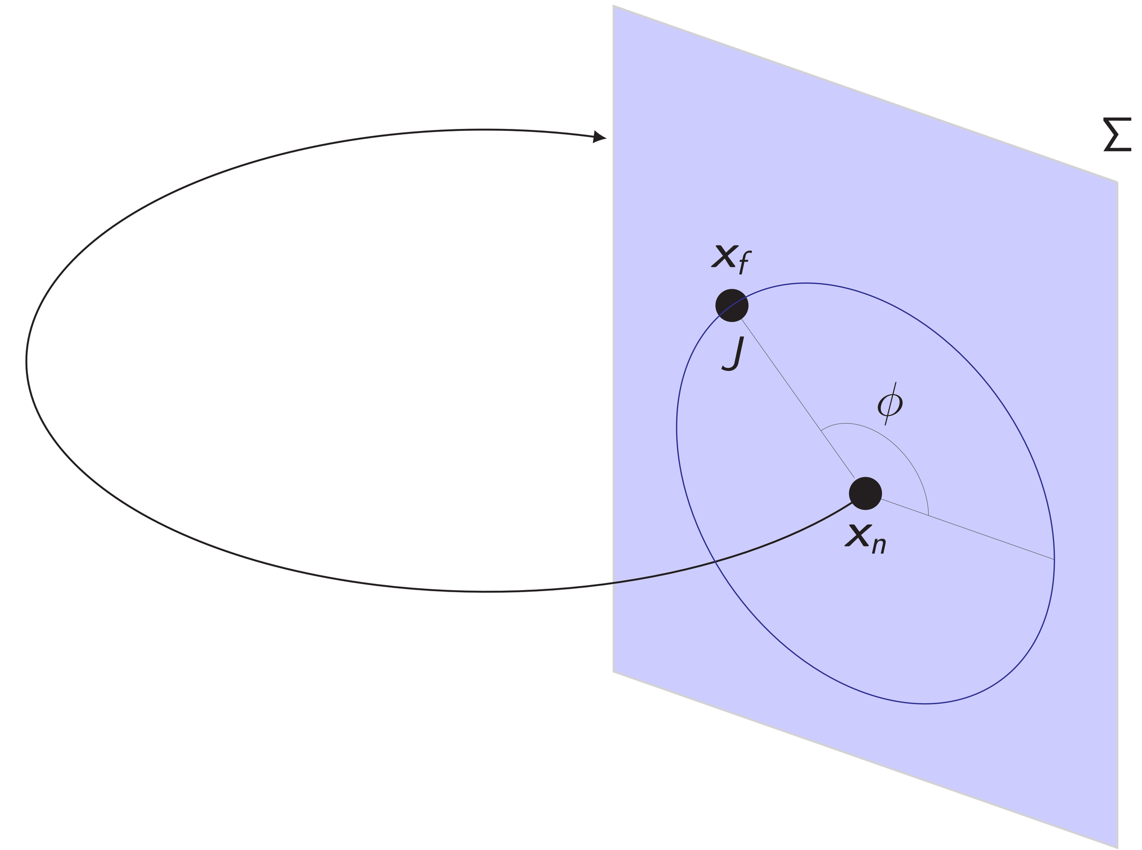


(a) Equatorial Transfer Trajectory



3D Transfer Trajectory

Reachability on the Poincaré section



Reachability set on a Poincaré section

- Poincaré map is a useful tool in the analysis of dynamical systems
  - Enables visualization of complicated systems - intrinsic structure becomes visible to the engineer
  - Rather than considering the entire state (6D position and velocity) we simply investigate the intersections with a lower dimensional space
  - This reduces the complexity of analyzing the dynamics and allows for visualization of highly complex dynamic interactions
- A periodic orbit on the Poincaré map is identified by fixed points x_n
- Using the low-thrust propulsion system of the spacecraft we can enlarge the space that is achievable
  - Reachability Set - the set of states which are attainable subject to the constraints of the system
  - The thruster of the spacecraft is used to design a transfer trajectory by repeatedly maximizing the reachability set
  - Thruster allows us to depart from the fixed orbit and intersect at a new state x_f
- Reachability Set is computed on the Poincaré section and provides additional insight
  - Spacecraft can only move to areas inside of the reachable set

Conclusions

- Demonstrate a transfer around an asteroid using multiple reachability sets
  - Each reachability set moves the spacecraft towards the target
- Alleviates the need for selecting accurate initial guesses
- Automatically gain insight into the feasible region of motion for the spacecraft
- Future work will extend this principle to landing trajectories on asteroids
  - Irregular shape of asteroids requires innovative techniques for controlling both position and orientation
  - Nonlinear control allows for the exploitation of the coupled dynamics
  - Complex dynamics requires accurate integration schemes - Variational Integrators
- Successful extension of previous work in the circular restricted three-body problem